Math 5630/6630 **Fall 2024**

Homework 0

Tags: Warm up *Due Date: 09/05/2024 11:59PM CST*

1 Homework Problems

This part of the homework assignment should be submitted via Canvas. You can scan the answers into PDF files, or typeset them in Word or LaTeX, then convert/compile them into PDF files.

Problem 1.1. *Calculate the first 4 terms of the Taylor series for functions*

(a). $f(x) = e^{\alpha x}$ *at point* $x_0 = \beta$.

(b). $f(x) = \frac{1}{x-\alpha}$ *at point* $x_0 = \beta$, *assume* $\beta \neq \alpha$.

Problem 1.2. *Estimate* $\max_{x\in[-\pi/4,\pi/4]} |f(x) - g(x)|$ *using the remainder formula of Taylor series.*

(a). $f(x) = \cos x$, $g(x) = 1 - \frac{1}{2}x^2$.

(b). $f(x) = \tan(x)$, $g(x) = x + \frac{x^3}{3}$.

Problem 1.3. *Apply the correct big* $\mathcal{O}$, Ω, Θ *notations. Example:* $n^1 = \mathcal{O}(n^2), \quad n \to \infty$.

$n^2 = \quad (n^{2.5}), \quad n \to \infty.$

$|\log n| = \quad (n^{1/3}), \quad n \to \infty.$

$|\log n|^2 = \quad (n^{1/n}), \quad n \to \infty.$

$\csc(n^{-1}) = \quad (\cot(n^{-1})), \quad n \to \infty.$

Problem 1.4. *Briefly explain why* $f(x) = x^4 - 3x^2 + 7x - 6$ *has a root inside* $[1, 2]$ *and* $[-3, -2]$. *Do not use any graphs.*

Problem 1.5. *Apply mean-value theorem to show*

$$|\cos(x/2) - \cos(y/2)| \leq \frac{1}{2}|x - y|, \quad x, y \in \mathbb{R}$$

Problem 1.6. *Let* $n > 0$, *show the maximum of* $f(x) = (1 - nx)(1 + x)^n$ *on* $[0, \frac{1}{n}]$ *is* 1. *Use that to show*

$$\frac{nx}{1 - nx} \geq (1 + x)^n - 1, \quad \forall x \in [0, \frac{1}{n}].$$

2 Programming Problems

Implement the following program tasks using your favorite programming language. The `Python` or `MATLAB` starter kit is available at GitHub Link. Follow the guidelines there for your submission.

Problem 2.1. *Write a program to compute the term a_m in the sequence $\{a_n\}_{n\geq 0}$:*

$$a_0 = 0, \quad a_1 = 1, \quad a_2 = 1, \quad a_{n+1} = a_n + a_{n-1} + a_{n-2}$$

Problem 2.2. *Write a program to use Laplace expansion (see its definition on page 2) to compute the determinant of a matrix $A \in \mathbb{R}^{n\times n}$, where $1 \leq n \leq 10$.*

Problem 2.3. *Write a program with running time [1] as close to 1 second as possible.*

[1] Here the "running time" refers to the wall-clock time, see its definition at Stackoverflow.